An Approach Combining Theory, Selection and Empirics Provides Evidence of Regularities in the Bias of Observational Methods

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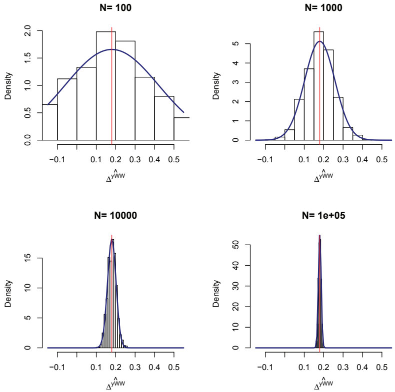
Observational Methods are unreliable

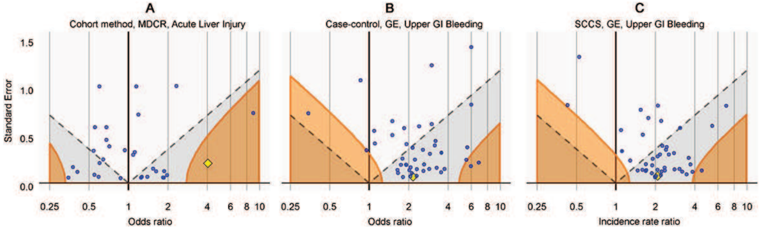
- Reliable Observational Methods would enable causal inference at a low cost
- Problem: Observational Methods are unreliable because their bias is unknown
- The bias of RCTs is much better known



Making Observational Methods reliable

Characterizing the distribution of the bias of Observational Methods would make them more reliable

- Correct confidence intervals including uncertainty about bias
- Choice of method ex ante

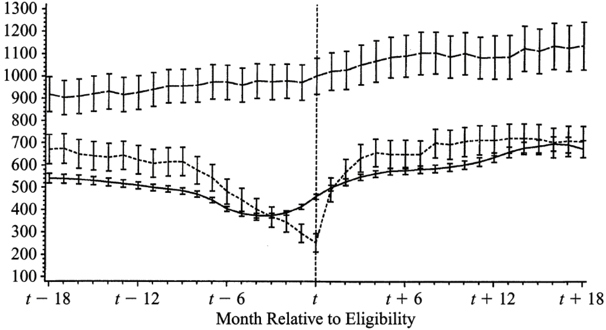


My proposal: combining 3 steps

1. Derive general properties using stylized theoretical models
2. Derive quantitative properties using simulations of calibrated models
3. Estimate the bias of Observational Methods from real data

An example: DID Matching and JTPs

- DID Matching emerges as least biased method when compared with RCTs
- Intuitive explanation: DID captures fixed effects, Matching captures transitory shocks



Why does DID Matching work?

1. Theoretical results
   - Intuitive story is wrong
   - Fallacy of alignment bias
   - Symmetric DID undoes time varying selection bias
2. Simulations
   - Fallacy of alignment bias is sizable
   - Symmetric DID resists to failure of symmetry
3. Empirical results confirm predictions

Literature

- Overall Approach: Chabé-Ferret (2015), Hill (2008), Eckles and Bakshy (2017)
- Theory: Heckman (1978), Nickell (1981), Heckman and Robb (1985)
- Simulations: Heckman, LaLonde, and Smith (1999), Huber, Lechner, and Wunsch (2013), Hatfield and Daw (2018)
- Empirics:
  - Within Study Comparisons: LaLonde (1986), Fraker and Maynard (1987), Glazerman et al (2003), Wong et al (2017), Chabé-Ferret et al (2018)
  - Sensitivity analysis: Ashenfelter and Card (1985), Anderson et al (2013), Caliendo et al (2014)
  - Meta-Analysis: Benson and Hartz (2000), Hemkens et al (2016)
  - Placebo-based approach: Schuemie et al (2013,2018)

Outline of the talk

Theoretical results

Simulations

Experimental evidence

Conclusion

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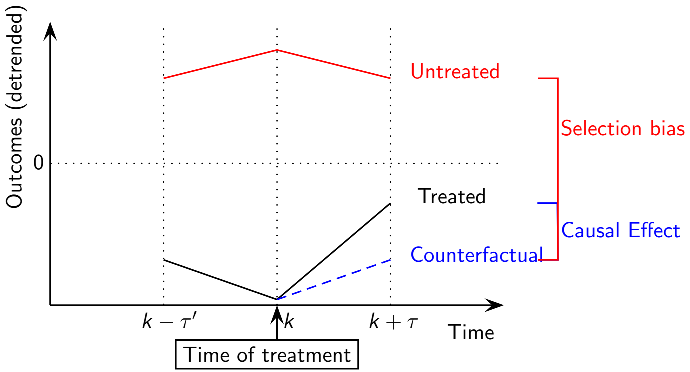
The model

$$Y^0_{i,t} = \delta_t + \mu_i + U_{it}$$
$$\text{with } U_{i,t} = \rho U_{i,t-1} + v_{i,t}$$
$$D^*_{i,k} = \theta_i + \gamma Y^0_{i,k-1} + \gamma^f v_{i,k}$$
$$D_{i,t} = \mathbb{1}\left[t \geq k\right] \mathbb{1}\left[D^*_{i,k} \geq 0\right]$$
$$\mathbb{E}\left[Y^0_{i,t} | D^*_{i,k}, Y_{i,k-1}\right] \text{ linear}$$
$$\text{Corr}(\mu_i, \theta_i) = \rho_{\theta,\mu}$$

- Both permanent (when $\rho_{\theta,\mu} \neq 0$) and transitory (when $\gamma \neq 0$ and $\rho \neq 0$) confounders
- Both limited (when $\gamma^f = 0$) and full (when $\gamma^f = \frac{\gamma}{\rho}$) information
- Both self-selection in a JTP and eligibility criteria

The estimators and their asymptotic biases



Estimators Bias

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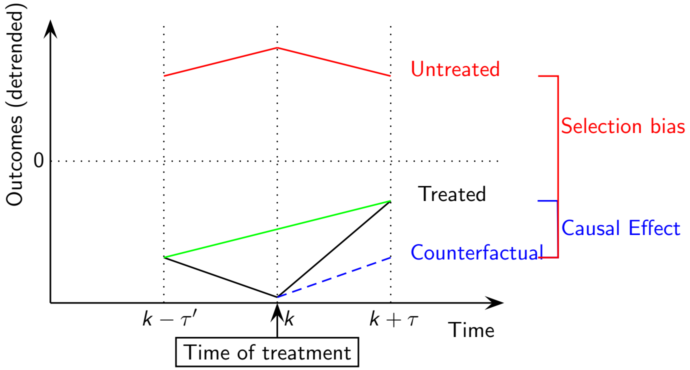

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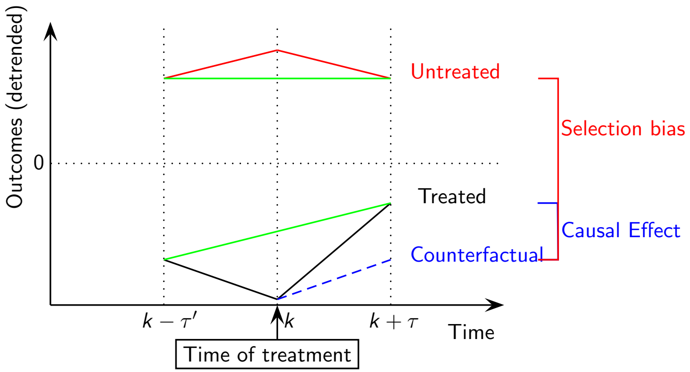

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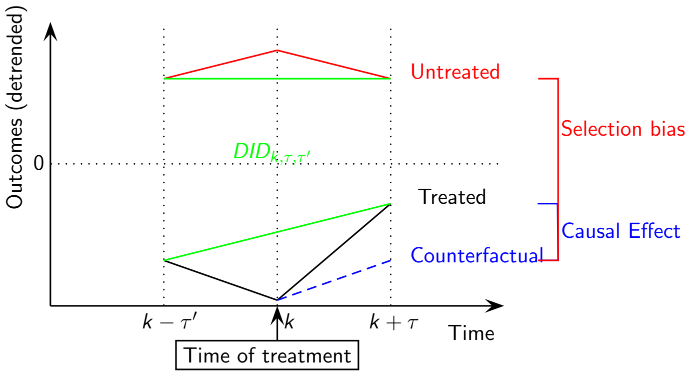

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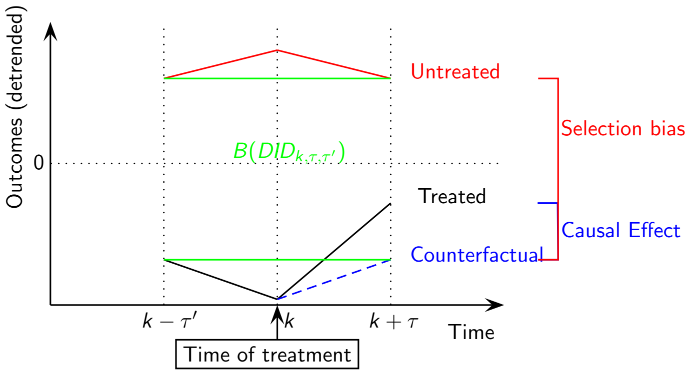

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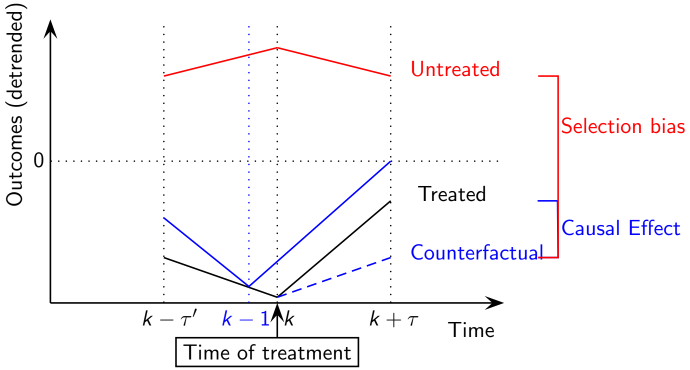

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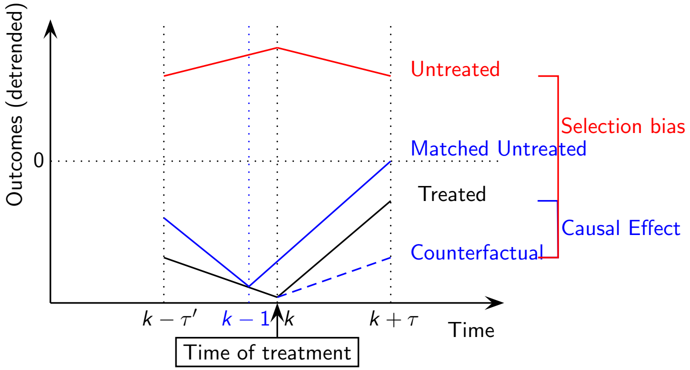

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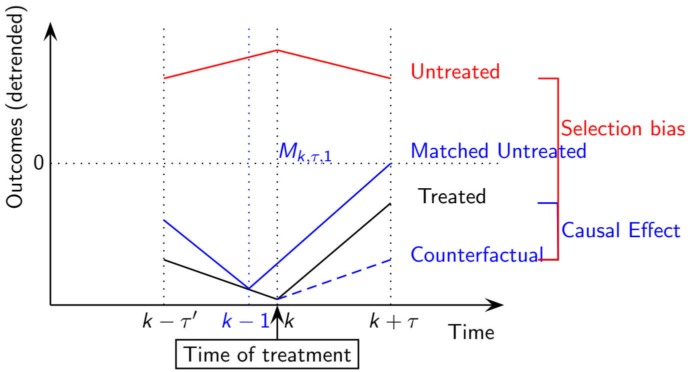

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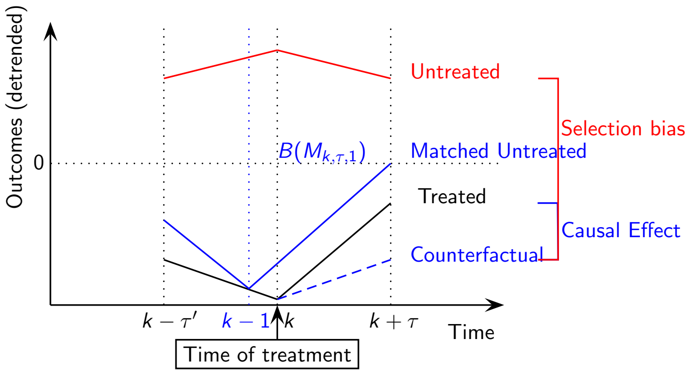

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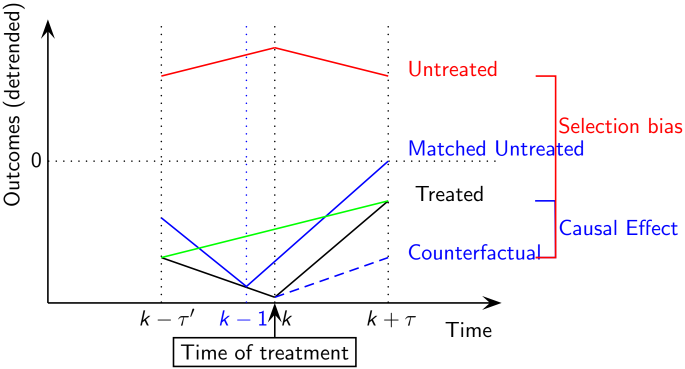

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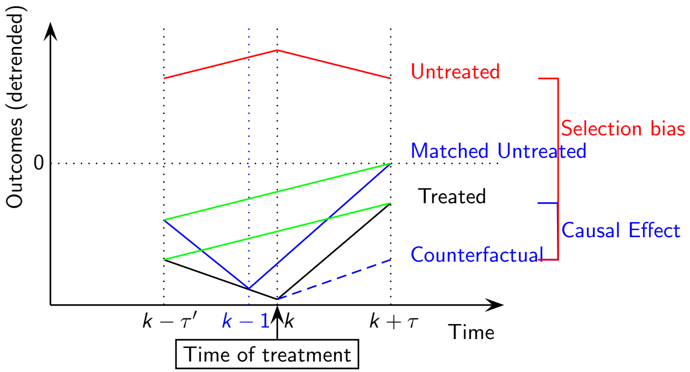

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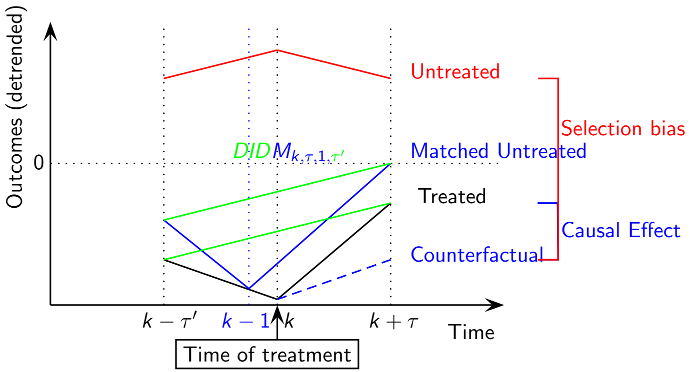

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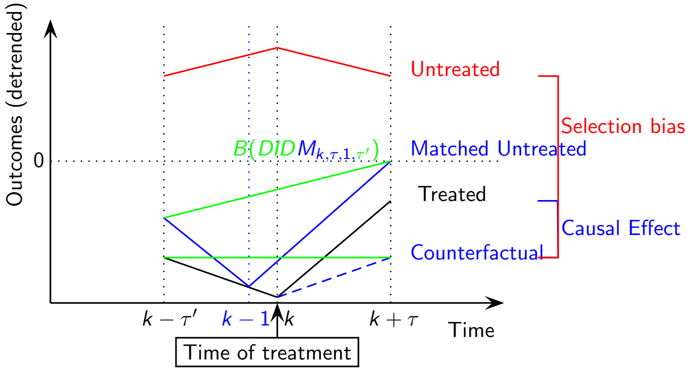


Estimators Bias

Conditions for consistency

Definition (Consistency)

An estimator $E_{k,\tau,\tau'}$ is consistent $\Leftrightarrow B(E_{k,\tau,\tau'}) = 0$,

- $\forall k \geq 3$
- $\forall \tau \geq 1$
- For $\tau' \in [2 \ldots \max\{2, k-1\}]$ with either:
  - $\forall \tau'$
  - For $\tau' = f(\tau)$

Main theoretical result

Theorem (Consistency of M, DID and DIDM)

$\forall k \geq 3$, $\forall \tau \geq 1$, *for* $\tau' \in [2 \ldots \max\{2, k-1\}]$,

(i) $B(M_{k,\tau,1}) = 0 \Leftrightarrow \rho_{\theta,\mu} = \gamma^f = 0$ *or* $\rho_{\theta,\mu} = \rho = 0$.

(ii) $B(DID_{k,\tau,\tau'}) = 0 \Leftrightarrow$

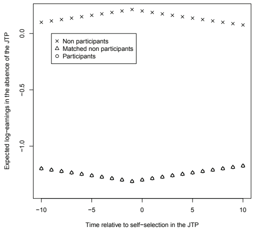
$$\begin{cases} \rho = 0 \text{ or } \gamma = \gamma^f = 0 \\ \text{or} \\ \sigma^2_{U_0} = \frac{\sigma^2}{1-\rho^2} \text{ and } \tau' = \tau + 1 + \frac{\ln \bar{\rho}}{\ln \rho}, \text{ with } \bar{\rho} = \rho + \frac{\gamma^f}{\gamma}(1-\rho^2). \end{cases}$$

(iii) $B(DIDM_{k,\tau,1,\tau'}) = 0 \Leftrightarrow$

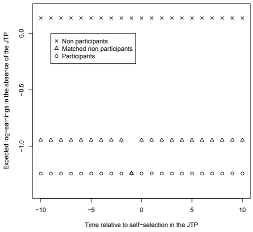
$$\begin{cases} \rho_{\theta,\mu} = \gamma^f = 0 \text{ or } \rho = 0 \\ \text{or} \\ \sigma^2_{U_0} = \frac{\sigma^2}{1-\rho^2} \text{ and } \tau' = \tau + 1 + \frac{\ln \rho^*}{\ln \rho}, \text{ with } \rho^* = \rho - \gamma^f \sigma^2 \frac{\frac{\sigma^2}{1-\rho^2} + \sigma^2_\mu}{\rho \sigma_\mu a \frac{\sigma^2}{1-\rho^2}}. \end{cases}$$

Matching Proof | DID Proof | DID Matching Proof

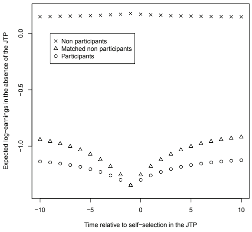
Illustration



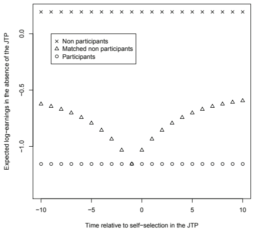
(a) $\rho_{\theta,\mu} = \gamma^f = 0$



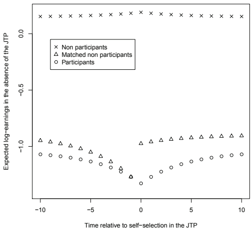
(b) $\gamma = \gamma^f = \rho = 0$



(c) $\rho_{\theta,\mu}, \rho, \gamma \neq 0$, $\gamma^f = 0$



(d) $\rho \neq 0$, $\gamma = \gamma^f = 0$



(e) $\rho_{\theta,\mu}, \rho, \gamma \neq 0$, $\gamma^f = \frac{\gamma}{\rho}$

Outline of the talk

Theoretical results

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The model

$$Y^0_{i,t} = a + b\frac{18+t}{10} + c\left(\frac{18+t}{10}\right)^2 + (\delta + r_t d)E_i + \mu_i + \beta_i t + U_{it}$$
$$\text{with } U_{i,t} = \rho U_{i,t-1} + m_1 v_{i,t-1} + m_2 v_{i,t-2} + v_{i,t}$$

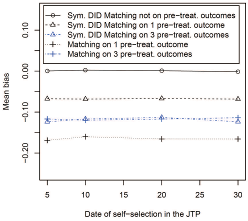
$$D^{*\iota}_{i,k} = \frac{\alpha_i}{r} - c_{i,k} - \mathbb{E}\left[Y^0_{i,k} | \mathcal{I}^{\iota}_{i,k}\right]$$
$$with\ c_{i,k} = c_i + \beta_x E_i - \left(a + b\frac{18+k}{10} + c\left(\frac{18+k}{10}\right)^2 + (\delta + r_k d)E_i\right)$$

- $\iota = f$: full information
- $\iota = l$: limited information
- $\iota = b$: Bayesian updating

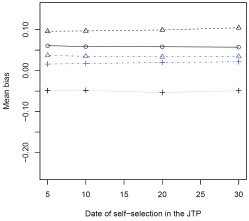
Parameterization

	RIP (MaCurdy, 1982)	HIP (Guvenen, 2007, 2009)
ρ	0.99	0.821
m_1	-0.4	0
m_2	-0.1	0
σ^2	0.055	0.055
σ_μ^2	0	0.022
σ_β^2	0	0.00038
$\sigma_{\mu,\beta}$	0	-0.002

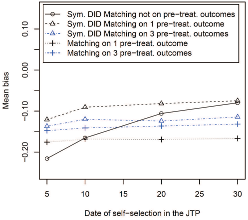
Results: RIP



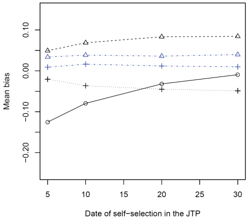
(a) Full Info, Long Run



(b) Limited Info, Long Run

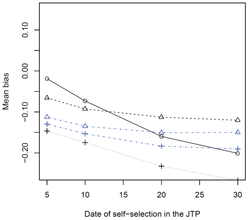


(c) Full Info, Short Run



(d) Limited Info, Short Run

Results: HIP



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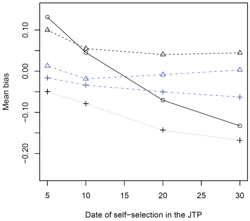
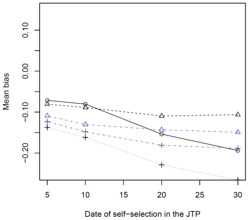


(b) Bayes, Long Run



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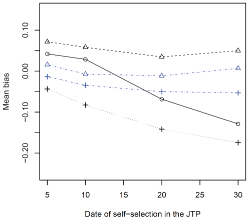

(d) Bayes, Short Run

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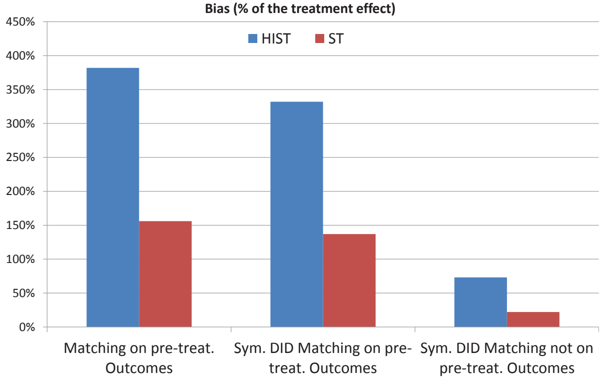
Evidence on the bias of observational methods

- Within Study Comparisons
- Between Study Comparisons
- Within Study Sensitivity Analysis

Results of Within Study Comparisons for JTPs



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Meta-analysis of Within Study Comparisons for JTPs

Glazerman, Levy, and Myers (2003)

RESULTS SHOWING THE EFFECT OF NONEXPERIMENTAL APPROACH ON BIAS IN EARNINGS IMPACTS

	Model Specification					
Explanatory Variable	1	2	3	4	5	6
Intercept	4,467*** (657)		4,687***(1,231)		5,775*** (1,120)	
Statistical method						
Regression	–1,583** (729)	–1,516** (705)	–1,476** (675)	–1,416** (706)	–3,225*** (1,195)	–3,572*** (1,284)
Matching	–478* (715)	–1,268** (794)	–807* (692)	–1,427*** (799)	–2,463* (1,375)	–3,178** (1,508)
Regression × Matching			951 (1,422)	1,320 (1,484)		
Difference-in-Differences	–1,874 (763)	–1,596 (816)	–1,859 (718)	–1,568 (813)	–3,532*** (1,253)	–3,231** (1,336)
Regression × Differences-in-Differences				2,325 (1,455)	2,676* (1,600)	
Matching × Differences-in-Differences					1,889 (1,477)	1,774 (1,547)
Selection correction	2,508* (1,248)	2,376 (1,305)	4,619 (1,048)	2,441 (1,299)	3,291*** (1,163)	3,072** (1,284)
Comparison group strategy						
Geographic match			–387 (973)	–646 (1,182)	–673 (957)	–581 (1,160)
National data set			1,145 (1,062)	1,695 (1,536)	915 (1,043)	1,668 (1,479)
Control group from another site			–1,762 (1,011)	NA	–2,124** (995)	–1,346 (2,863)
Study dummies included	No	Yes	No	Yes	No	Yes

NOTE: Dependent variable is the absolute value of the bias in annual earnings, expressed in 1996 dollars. Standard errors are in parentheses. All explanatory variables are dummy variables. Sample size is 69 bias estimate types.

*Significantly different from zero at the .10 level. **Significantly different from zero at the .05 level. ***Significantly different from zero at the .01 level, two-tailed test.

Between Study Analysis of JTPs (Example)

Card, Kluve, and Weber (2015)

	Number Est's. (1)	Median Sample Size (2)	Percent RCT's (3)	Mean Program Effect on Prob. Emp. (×100) Short Term (4)	Medium Term (5)	Longer Term (6)
By Evaluation Design:						
Experimental	166	1,471	100.0	4.4	2.5	0.5
				(28)	(25)	(15)
Non-experimental	691	16,000	0.0	0.9	6.0	11.0
				(113)	(118)	(53)

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Conclusions so far

- The proposed approach seems to work
- Combining DID and Matching on pre-treatment outcomes is not a great idea

To do for this paper

- Simulations with levels
- Re-analyze JTPA data
- Nickell bias?

Further research

1. Bias of OM for JTPs
   - One prediction to be tested: controlling for more pre-treatment outcomes could make things better
   - Two empirical results to explain
     - Conditioning for labor market transitions improves matching
     - The bias of observational methods increases with time after treatment
2. Bias of OM in other applications
   - Collect more estimates of bias
     - Within Study Comparisons using RCTs with imperfect compliance (Chabé-Ferret et al, 2018)
     - Test validity of pseudo-experiments
   - Develop simulations and theories
   - Put information into accessible database (SKY: Social Science Knowledge Accumulation Initiative)

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Estimators: definition

$$
\begin{aligned}
M_{k,\tau,1} &= \mathbb{E}\left[\mathbb{E}\left[Y_{i,k+\tau}|D_{i,k}=1, Y_{i,k-1}\right]\right. \\
&\quad \left. - \mathbb{E}\left[Y_{i,k+\tau}|D_{i,k}=0, Y_{i,k-1}\right]|D_{i,k}=1\right] \\
DID_{k,\tau,\tau'} &= \mathbb{E}\left[Y_{i,k+\tau} - Y_{i,k-\tau'}|D_{i,k}=1\right] \\
&\quad - \mathbb{E}\left[Y_{i,k+\tau} - Y_{i,k-\tau'}|D_{i,k}=0\right] \\
DIDM_{k,\tau,1,\tau'} &= \mathbb{E}\left[\mathbb{E}\left[Y_{i,k+\tau} - Y_{i,k-\tau'}|D_{i,k}=1, Y_{i,k-1}\right]\right. \\
&\quad \left. - \mathbb{E}\left[Y_{i,k+\tau} - Y_{i,k-\tau'}|D_{i,k}=0, Y_{i,k-1}\right]|D_{i,k}=1\right].
\end{aligned}
$$

Back

Bias: definition

$$
\begin{aligned}
B(M_{k,\tau,1}) &= \mathbb{E}\left[\mathbb{E}\left[Y^0_{i,k+\tau}|D_{i,k}=1, Y^0_{i,k-1}\right]\right.\\
&\quad \left.-\ \mathbb{E}\left[Y^0_{i,k+\tau}|D_{i,k}=0, Y_{i,k-1}\right]|D_{i,k}=1\right]\\
B(DID_{k,\tau,\tau'}) &= \mathbb{E}\left[Y^0_{i,k+\tau}-Y^0_{i,k-\tau'}|D_{i,k}=1\right]\\
&\quad -\ \mathbb{E}\left[Y^0_{i,k+\tau}-Y^0_{i,k-\tau'}|D_{i,k}=0\right]\\
B(DIDM_{k,\tau,1,\tau'}) &= \mathbb{E}\left[\mathbb{E}\left[Y^0_{i,k+\tau}-Y^0_{i,k-\tau'}|D_{i,k}=1, Y_{i,k-1}\right]\right.\\
&\quad \left.-\ \mathbb{E}\left[Y^0_{i,k+\tau}-Y_{i,k-\tau'}|D_{i,k}=0, Y_{i,k-1}\right]|D_{i,k}=1\right]
\end{aligned}
$$

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Consistency of Matching: sketch of proof

By linearity of conditional expectations:

$$\mathbb{E}\left[Y_{i,t}^0 | D_{i,k}^*, Y_{k-1}^0\right] = \mathbb{E}\left[Y_{i,t}^0\right] + \theta_{Y_{k+\tau}^0, D_k^*}\left(D_{i,k}^* - \mathbb{E}\left[D_{i,k}^*\right]\right) + \theta_{Y_{k+\tau}^0, Y_{k-1}^0}\left(Y_{i,k-1}^0 - \mathbb{E}\left[Y_{i,k-1}^0\right]\right)$$

$$\theta_{Y_{k+\tau}^0, D_k^*} = \frac{\overbrace{\sigma_{Y_{k+\tau}, D_k^*}\sigma^2_{Y_{k-1}} - \sigma_{Y_{k-1}, D_k^*}\sigma_{Y_{k-1}, Y_{k+\tau}}}^{\text{num}_{k,\tau}}}{\sigma^2_{D_k^*}\sigma^2_{Y_{k-1}} - \sigma^2_{Y_{k-1}, D_k^*}}$$

$$B(M_{k,\tau,1}) = \theta_{Y_{k+\tau}^0, D_k^*}\mathbb{E}\left[\mathbb{E}\left[D_{i,k}^* | D_{i,k} = 1, Y_{k-1}^0\right] - \mathbb{E}\left[D_{i,k}^* | D_{i,k} = 0, Y_{k-1}^0\right]\right]$$

$$\text{num}_{k,\tau} = \rho^\tau \underbrace{\left(\gamma^f\sigma^2\left[\sigma^2_\mu + \sigma^2_{U_{k-1}}\right] - \rho_{\theta,\mu}\sigma_\theta\sigma_\mu\rho\sigma^2_{U_{k-1}}\right)}_{F(k)} + \underbrace{\rho_{\theta,\mu}\sigma_\theta\sigma_\mu\sigma^2_{U_{k-1}}}_{G(k)}$$

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Consistency of DID Matching: sketch of proof

$$B(DIDM_{k,\tau,1,\tau'}) = 0 \Leftrightarrow \text{num}_{k,\tau} - \text{num}_{k,-\tau'} = 0$$

$$\text{num}_{k,\tau} - \text{num}_{k,-\tau'} = B(\tau,\tau') + \rho^{2(k-\tau')} C(\tau,\tau')$$

$$B(\tau,\tau') = H(\tau') + \rho^{\tau} I$$

$$H(\tau') = \sigma_\mu \rho_{\theta,\mu} \sigma_\theta \rho^{\tau'-1} \frac{\sigma^2}{1-\rho^2}$$

$$I = \frac{\sigma^2}{1-\rho^2}(\gamma^f \sigma^2 - \rho \sigma_\mu \rho_{\theta,\mu} \sigma_\theta) + \gamma^f \sigma^2 \sigma_\mu^2$$

$$C(\tau,\tau') = J(\tau') + \rho^{\tau} K(\tau')$$

$$J(\tau') = \sigma_\mu \rho_{\theta,\mu} \sigma_\theta \rho^{\tau'-1} \left(\sigma_{U_0}^2 - \frac{\sigma^2}{1-\rho^2}\right)$$

$$K(\tau') = \gamma^f \sigma^2 - \left(\sigma_{U_0}^2 - \frac{\sigma^2}{1-\rho^2}\right) \sigma_\mu \rho_{\theta,\mu} \sigma_\theta \rho$$

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Consistency of DID Matching: sketch of proof (cont'd)

$$
\begin{aligned}
B(\tau, f(\tau)) &= \rho^{\tau} L(\tau) \\
L(\tau) &= \rho \sigma_{\mu} \rho_{\theta,\mu} \sigma_{\theta} \frac{\sigma^2}{1-\rho^2} (\rho^{f(\tau)-\tau-2} - 1) + \gamma^{f} \sigma^2 \left(\frac{\sigma^2}{1-\rho^2} + \sigma_{\mu}^2 \right) \\
C(\tau, f(\tau)) &= \rho^{\tau+2(f(\tau)-1)} M(\tau) \\
M(\tau) &= \left(\sigma_{U_0}^2 - \frac{\sigma^2}{1-\rho^2} \right) N(\tau) \\
N(\tau) &= \sigma_{\mu} \rho_{\theta,\mu} \sigma_{\theta} \rho (\rho^{-(f(\tau)+\tau)} - 1) + \gamma^{f} \sigma^2
\end{aligned}
$$

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Consistency of DID: sketch of proof

$$B(DID_{k,\tau,\tau'}) = 0 \Leftrightarrow \sigma_{Y^0_{k+\tau},D^*_k} - \sigma_{Y^0_{k-\tau'},D^*_k} = 0$$

$$\sigma_{Y^0_{k+\tau},D^*_k} - \sigma_{Y^0_{k-\tau'},D^*_k} = P(\tau,\tau') + \rho^{2(k-\tau')}Q(\tau,\tau')$$

$$P(\tau,\tau') = \gamma(\rho^{\tau+1} - \rho^{\tau'-1})\frac{\sigma^2}{1-\rho^2} + \gamma^f\rho^\tau\sigma^2$$

$$Q(\tau,\tau') = \gamma\left(\sigma^2_{U_0} - \frac{\sigma^2}{1-\rho^2}\right)(\rho^{\tau+1}\rho^{2(\tau'-1)} - \rho^{\tau'-1})$$

$$P(\tau,f(\tau)) = \rho^{\tau+1}\left(\gamma\rho\frac{\sigma^2}{1-\rho^2} + \gamma^f\sigma^2 - \gamma\rho^{f(\tau)-\tau-1}\frac{\sigma^2}{1-\rho^2}\right)$$

$$Q(\tau,f(\tau)) = \rho^{\tau+1+2(f(\tau)-1)}\gamma\left(\sigma^2_{U_0} - \frac{\sigma^2}{1-\rho^2}\right)\left(1 - \rho^{-f(\tau)-\tau+1}\right).$$

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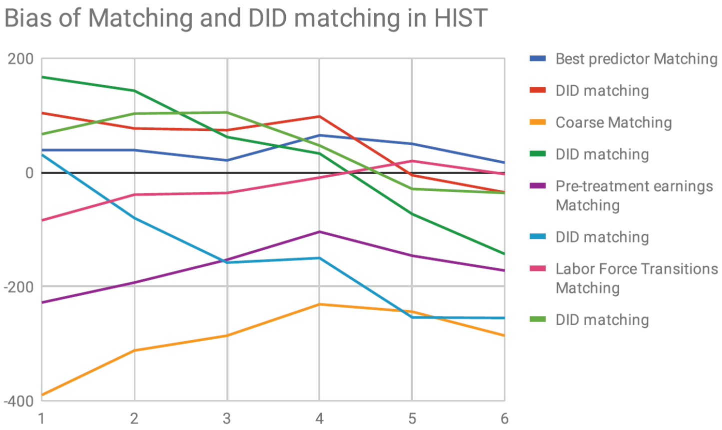
Results of HIST



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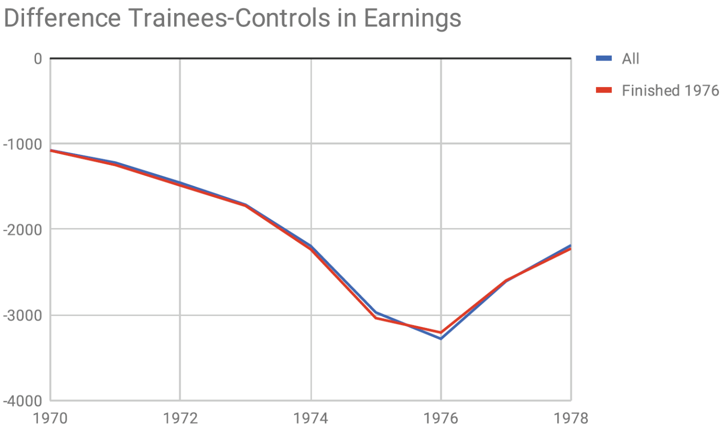
Results of Ashenfelter and Card



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