Money and the auctioneer:
Assessing Walras's Decentralised Exchange Process[*]

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> *Si nous voulons parvenir à la connaissance des lois qui régissent l'échange des biens, il est nécessaire que nous remontions d'abord aux motifs qui font agir les hommes dans l'échange des biens, aux faits indépendant de la volonté de l'échangeur, qui sont avec l'échange des biens dans un rapport causal.*
> (Carl Menger's letter to Léon Walras, February 1884[1])

Abstract

Recent works on Walras have underlined a part of this author's theory that had passed unnoticed, i.e. the role he gave to money as a device allowing the decentralised allocation of goods. This contradicts, at least partially, the shared idea among today's monetary theorists who point out the difficulty to introduce money in the perfect competition framework *à la* Arrow-Debreu. In this paper, after having reviewed the logical construction of Walras' price theory, we use modern monetary theory to examine Walras' justification of the use of money as a medium of exchange. This allows us to isolate the conditions under which this introduction can lead effectively to the circulation of the medium of exchange and to discuss the consistency of the assumptions made to introduce money with those of the competitive aspects of his theory.

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[1] In Antonelli (1953), p. 281.

1 Introduction

Modern theory of money has experienced a deep change in the last thirty years. Most of the models built after the 1970's can be understood as a return to the function of money as a medium of exchange. The methodology of modern monetary economics consists in constructing exchange environments in which money has an essential role as medium of exchange. This new literature in the theory of money[2] responds to what has been presented as the failure of the General Equilibrium model of the neo-Walrasian tradition to integrate money in the theory of value. However, as we shall try to show, the neo-Walrasian tradition (mainly developed by Patinkin and the Overlapping Generations models) does not address the monetary question as Walras did.

Contrary to the traditional view on Walras's theory, one can assert that Walras analyses money as medium of exchange and not as store of value. This characteristic of his monetary analysis has been hidden by the predominance of the General Equilibrium Model *à la* Arrow and Debreu. Therefore, it is important to discuss the method Walras uses when he introduces money into his framework. Walras proceeds in two steps: First, he assumes an extremely organised market in which he deals with the formation of prices. Then, he proposes the integration of money into this model as the solution to a decentralised exchange process. This asymmetric treatment of price formation and exchange makes it difficult to know whether he was successful in the integration of Money and Value theories. Two competing interpretations can be proposed. On the one hand, the integration of money can be seen as an extension of his value theory to an object called money, without incidence on the price theory. On the other hand, money can be seen as a necessary condition for the realisation of the general equilibrium allocation. The later has been rarely being adopted[3]. Nevertheless, this interpretation maybe a way to understand why Walras's pure monetary theory was considerably modified through the different editions of his *Elements*[4].

In this paper we propose to explore two issues derived from the analysis of Walras's monetary theory from the modern theory point of view. First, we propose a critical lecture of the evolution of the neo-Walrasian tradition on the integration of money in the general equilibrium model. The common complaint of the 1970s was that the Walrasian framework

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[2] For a classical survey on the first models in this line see Ostroy and Starr (1991). However, this reference does not contain the most recent developments on the 'money as a medium of exchange' approach. For a recent survey, containing in particular the search monetary approach, see Wallace (2001).
[3] With the important exception of Rebeyrol (1998 and 1999).
[4] *Element d'économie politique pure* (Walras 1988).

has no place for a decentralised exchange process. This perception was mainly founded upon the existence of centralised institution put in charge to accomplish the final allocations within the neo-Walrasian model (i.e. Debreu's (1959) *clearing house* or Arrow and Hahn's (1971) *auctioneer*). As we shall show, this hypothesis is not present within Walras's pure theory. This is even the case for the first 28 lessons where money is clearly absent. Contrary to the neo-Walrasian tradition, Walras actually deals with the decentralised exchange problem. However, his treatment of the problem is not as neat as one would like to be.

The analysis of Walras's shortcomings concerning his pure monetary theory (as in Lesson 29 of the *Elements*'s last edition) leads us to the second main issue of this paper. Following recent monetary theory, we try to reappraise Walras's attempt to integrate a decentralised exchange process within his perfect competition framework. We show that even if this attempt goes beyond the traditional analysis of money as store of value of the Neo-walrasian models, Walras's model is not a completely satisfying answer if it is to be judged from the 1990s-2000s monetary theory. We argue that as the problem of a decentralised exchange process becomes the main issue of monetary theory, monetary theory adopts a new criterion in order to judge the well-founded of the integration of money.

This new criterion leads to analyse monetary exchange as an alternative arrangement to be compared with other forms of decentralised exchange. Money is no more an alternative to assets as a store of value, but an alternative to other exchange technologies. Walras's theory does not go this far in the analysis of the decentralised exchange process. Furthermore, we show that some aspects of Walras's pure competitive model show to be difficult to conciliate with the recent criterion of the integration of money as the choice of individuals' exchange strategies. Following this line of argument, monetary analysis may show how the monetary exchange can allow sustaining some final allocations which are not possible to be attained trough a different exchange technology. This is what Wallace (2002) calls the "essentiality of money".

The rest of the paper is organised as follows. Section 2 describes the key elements of Walras's model of perfect competition and equilibrium. We shall show how his theory of money is insufficient to justify the "essentiality of money" within his framework. Our argument proceeds in two steps. First, partially following Rebeyrol (1999) and Bauvert (2002), and contrary to the traditional view, we shall show that Walras actually attempted to analyse the exchange process (§3). We thus show the main difference between Walras's and the Neo-walrasian theory of market. In order to interpret Walras's attempt, we follow theorists of the seventies who depart from the Arrow-Debreu tradition by introducing a modification of

the perfect competition framework (§4). These changes are similar to Walras's own pure theory of money. However, more recent works can help to analyse the shortcomings of Walras's attempt, constituting an appeal to deepen the work on his monetary theory. Section 4 contains some final remarks and conclusions of the paper.

2   Perfect Competition and the Neutrality of Pure Economics

As has been clearly stated by Jaffé (1980: 530), Walras's pure economics is an integral part of a larger system of social philosophy. Walras tried to build a pure theory of economics as a neutral system with respect to the criteria of distributional justice. His pure theory of market, prices and money must satisfy commutative justice in order to describe production and exchange as free individual decisions isolated from distributional considerations. Agents may be free to exchange and produce within the limits of feasible allocations that respect individuals' budget constraints. These requirements lead Walras to impose a regime of free competition as the main (and sole) framework of his pure theory of economics.

Since the first versions of his pure theory of prices Walras intends to distinguish perfect competition from monetary exchange. In the following well known quotation from his 1873 lectures at the *Académie de sciences morales et politiques de Paris*, Walras presents money as a pure practical economic device, excluded from the pure theory of competition:

> Nous allons étudier le phénomène des prix se produisant dans ces conditions de concurrence supposées rigoureuses, en faisant abstraction des petites circonstances perturbatrices comme en physique, en mécanique, on fait abstraction tout d'abord, sauf à les introduire ensuite dans les formules, de la résistance des milieux, du frottement, etc. Seulement, nous écarterons aussi l'intervention de la monnaie. Pour avoir une idée exacte du mécanisme de la concurrence, il nous a bien fallu l'emprunter à l'un de ces marchés où se font des ventes et des achats de marchandises contre or et argent ; mais il est clair que l'intervention de la monnaie, qui est une simplification pratique, est une complication théorique qui doit être écartée. (Walras 1993 [1874]: 33)

The price taker behaviour and the "no-exchange-out-of-equilibrium" hypothesis are the main characteristics of Walras's perfect competition framework. This construction is founded on two elements: the numeraire and the fiction of a *tâtonnement* process. The former allows avoiding the treatment of strategic behaviours within a general equilibrium system and establishes a dichotomy between the exchange process and price formation. The latter prevents pure theory of prices from dealing with distributional effects of out-of-equilibrium exchanges.

2.1   The Numeraire and the Market Organisation

In the *Eléments d'économie politique pure* (henceforth *EEPP*) Walras presents the foundations of his theory of prices and competition in a two commodities pure-exchange framework (Section II, 4th edition). Money is absent from this analysis (it shall remain absent until section VI in the 4th edition). Within this framework the two commodities are directly exchanged in a common market and consequently there exists a unique rate of exchange (relative price). Even since the simplest case of exchange, Walras introduces the hypothesis of price-taking. In order to fulfil the commutative justice requirements agents exchange only at the equilibrium price of the market. As has been well noticed by Rebeyrol (1999: 100): « …l'absence de transaction en déséquilibre n'est une hypothèse exorbitante que si l'on admet l'absence de conscience du déséquilibre. ». This hypothesis is the reduced form of the hypothesis of common knowledge of all exchange opportunities. Even if Walras does not explicitly describe the details of the organisation of this two-commodities-market, the reference to the stock-market structure leads to conclude that this is not a disorganised bilateral exchange process.

The generalisation of the price theory to the n-commodities case strengthens the requirements of a well organised market process. In section III of his *EEPP* 4th edition Walras attempts to demonstrate a "true theorem of general equilibrium". A simple extension of the two-commodities framework leads to a system of one independent market for every couple of commodities (i.e. *n(n-1)/2* markets for *n* commodities). However, if nothing is said about the level of information of agents this can result in an "incoherent" system of prices. A price system is incoherent if different exchange strategies lead to different final allocations for an agent. In other words, if some arbitrage gains of indirect exchange are still possible.

The condition of a general equilibrium system of prices is resumed by Walras (1988) in *EEPP*:

> L'équilibre parfait ou général du marché n'a lieu que si le prix de deux marchandises quelconques l'une en l'autre est égal au rapport des prix de l'une et l'autre en une troisième quelconque. (161-163)

This condition is, yet again, a requirement of neutrality of the perfect competition solution. If there are un-exploited possibilities of arbitrage some agents lose exactly what other agents get by exchanging at those prices. The value of an individual's budget constraint is not independent of the order he follows in order to obtain his desired final allocation. And a generalised direct barter exchange within a system of isolated two-commodities markets may not necessarily result in a perfect general equilibrium situation (Walras 1988: 163). However,

if those possibilities of gains are common knowledge, the arbitrage strategies may be neutralised. Walras's solution to this problem is the introduction of a common unit of measure of individuals' budget constraints, namely the common numeraire. If prices are all quoted in terms of a common numeraire, agents can verify the coherence of the actual price system. Due to common knowledge hypothesis on prices, arbitrages are neutralised. In consequence Walras concludes that « … si on a crié des prix en numéraire, la condition d'équilibre général a été remplie ipso facto » (1988: 200).

The numeraire is thus more than a hypothesis aiming at simplifying the computation of a price system. The existence of a common measure of prices is accompanied of particular conception of the organisation of markets. The numeraire is the language necessary for the common knowledge hypothesis. The consequence of these hypotheses is a market system where strategic behaviours are absent[5] (because they have already been neutralised!).

2.2   The Tâtonnement Process and Price-taking Behaviour

As P. Bridel (1997 and 2002) has clearly stated, the evolution throughout the different chapters and editions of the *EEPP* of Walras's theory of price formation explained by a *tâtonnement* process has important consequences on his monetary theory. Because of Walras's attachment to the internal coherence of his pure economics, the theory of stability and dynamics is subordinated to the notion of neutrality of the exchange process. This implies that the process of price formation in perfect competition is finally presented as an instantaneous adjustment of all markets towards equilibrium.

The well-known discussion of the walrassian *tâtonnement* appears in the first editions of the *EEPP* as a description of an adjustment process that actually takes place in the market. Here, Walras underlines the role that competition plays in this process. The literary description of the process, which will prevail until the last edition of the *EEPP*, sustains that during the formation of equilibrium prices agents have an active role and that competition is the result of individual decisions:

> La valeur d'échange laissée à elle-même se produit naturellement sur le marché sous l'empire de la concurrence. Comme acheteurs, les échangeurs demandent à l'enchère, comme vendeurs, ils offrent au rabais, et leur concours amène ainsi une certaine valeur d'échange des marchandises tantôt ascendante, tantôt descendante

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[5] Costa (1988 and 2002) also analyses the problems Walras faces trying to avoid the arbitrage questions. He concludes that the analytical difficulties associated with those problems lead Walras to avoid an explicit treatment of competition and exchange. Following Rebeyrol (1999), we rather argue that it was not a matter of difficulty but a more profound attachment to a particular conception of pure economics.

et tantôt stationnaire. Selon que cette concurrence fonctionne plus ou moins bien, la valeur d'échange se produit d'une manière plus ou moins rigoureuse.

<div align="right">(Walras 1988: 70)</div>

One question arises after reading this passage: What does a more or less rigorous competition mean? Walras answers in the same paragraph:

> Les marchés les mieux organisés sous le rapport de la concurrence sont ceux où les ventes et achats se font à la criée, par l'intermédiaire d'agents tels qu'agents de change, courtiers de commerce, crieurs, qui les centralisent, de telle sorte qu'aucun échange n'ait lieu sans que les conditions en soient annoncées et connues et sans que les vendeurs puissent aller au rabais et les acheteurs à l'enchère.

<div align="right">(Ibidem.)</div>

That is, if the process of price formation is to take place under perfect competition, without interfering with distributive justice, agents cannot exchange at prices different from those of equilibrium. In order to guarantee this result, market intermediaries centralise supplies and demands thus avoiding exchanges out of equilibrium. These intermediaries are stock brokers. Therefore, in spite of the initial assertion according to which agents are active during the price formation process, Walras ends up reducing perfect competition to a situation where agents are price-takers because the active part of the market is left to *crieurs* and *courtiers*. However, these agents appear only as intermediaries during the price formation process but nothing is said about their role within the actual exchange process.

This description of *tâtonnement*, which seems to appeal to an empirical argument in the text presented above, is clearer in Walras's lecture before the Académie de sciences morales et politiques de Paris read in 1873 (Walras 1993). Here Walras explains that to avoid any possible ambiguity and to guarantee coherence with the aim of his pure theory[6] he assumes *tâtonnement* takes place through the use of an automatic calculator (a computer) which determines the quantities offered and demanded by each individual and can finally calculate general equilibrium prices (Walras 1993). Walras thus avoids the problem of the consequences of disequilibrium on expectations (if the process takes place under an auctioneer) or on distributive effects (if exchanges take place in disequilibrium). As most of Walras's scholars since Jaffé's works (1965, 1980 and 1981) have remarked, the *tâtonnement* process is a process during which no exchanges take place and can be interpreted (see Bridel

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[6] After the first edition of *EEPP* (1874) and probably as a consequence of Bertrand's and Edgeworth's criticisms Walras makes more explicit his hypothesis of absence of exchange out of equilibrium. In spite of this textual evidence Walker (1990a and 1990b) considers, contrary to most scholars' interpretation, that it is wrong to assume that this hypothesis means Walras does not study the disequilibrium exchanges. Be this as it may, it is certain that as long as Walras's monetary theory is concerned, disequilibrium exchanges are absent. In lesson 29 of the *EEPP* 4th edition, "circulation" (exchange process) begins after equilibrium prices are quoted.

1997, chapter 4; 2002) as a method to solve a system of simultaneous equations whose roots are the equilibrium prices.

Finally, an important reason that completes and reinforces the requirements of commutative justice and explains why Walras focuses on the equilibrium situation is that it is optimal (i.e. Pareto optimal). If exchanges were to take place at disequilibrium prices there would be unsatisfied exchange possibilities that could induce an agent aware of this situation to propose a different price that would better his condition and would not worsen that of the person willing to exchange with him[7]. The normative[8] properties of general equilibrium are then imposed from the foundation of this theory as the main element economics would never give up. This is why the existence and welfare theorems can be dealt with independently from the dynamics. These theorems allow establishing a theoretical reference with regard to Walras's idea of commutative justice. Therefore, without demonstrating the stability of equilibrium Walras may assert, just as contemporary applied economics does, in his *Studies of applied political economy* [1898] that:

> Les éléments du système économique sont des services qui, sous le régime de la libre concurrence, *tendent naturellement* à se combiner en produits de la nature et de la quantité propres à donner la plus *grande satisfaction possible des besoins* dans les limites de cette double condition que chaque service comme chaque produit n'ait qu'un seul prix sur le marché et que le prix de vente de chaque produit soit égal à son prix de revient en services.

<div align="right">(Walras 1992: 77, our emphasis)</div>

Walras considers that this situation should be the general frame for a pure economic theory. If there is to be a pure theory of money, it must be adapted to the frame of competitive general equilibrium. Besides from being static with passive agents as regards prices and only focusing on equilibrium situations, this framework leaves no place for a theory of the organisation of the exchange process without taking into account the perturbations of the equilibrium so induced. However, Walras attempted, through the different editions of his *EEPP* and other writings (in particular Walras 1992 [1898]), to introduce a monetary theory

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[7] This reasoning may be easily understood from what is nowadays known as recontracting in a model à la Edgeworth. The famous debate between Walras and Edgeworth regarding the *tâtonnement* process leads to the conclusion that, although Walras does not accept the terms of Edgeworth's argument, he accepts that at least for the pure theory of prices the criteria of commutative justice and distributive neutrality imply that "Il est parfaitement juste de proposer immédiatement le prix unique" (Walras 1896: 184). For an in-depth discussion of this point see Rebeyrol (1999: 90-100). For an opposite point of view see Walker (1987).

[8] It is important to underline that here we are referring to the modern concept of "Pareto optimality" which has nothing to say in terms of distributive justice or even of first best results. Then we accept the critical point made by Jaffé (1977) against Hicks and Baumol concerning Walras's extremely optimistic view of a free market issue in terms of a social welfare criterion in terms of first best allocation or distributive justice.

aiming at demonstrate the neutrality of the monetary exchange vis-à-vis the equilibrium real prices.

3   The Integration of Money into Perfect Competition

3.1   An increased interest in taking into account the exchange process

Here we shall not give a detailed presentation of the evolution of Walras's theory of money[9]. We only want to underline how this evolution indicates that Walras takes more attention to the exchange technology. In fact, Walras's final version of his pure theory of money pushed him to explicitly deal with the actual exchange process and the difficulties of the direct barter exchange. However, the main goal of his monetary theory remained the same: to show the conditions of the stability of the value of money.

Walras frequently expresses some doubts concerning the introduction of the actual exchange process within pure economics. As it was showed above, up to his *Théorie de la monnaie* (Walras 1992 [1898]), including the second and third editions of the *EEPP*, he considered monetary exchange as a practical simplification and a theoretical difficulty (Walras 1988: 540 and 541).

In the first (1874) edition of the *EEPP* Walras's theory of money was limited to the formulation of a Fisherian like exchange equation: "la circulation à desservir". At this stage, nothing is said about the individual decisions concerning the money demand. This kind of considerations begin to appear in the second (1889) and third (1896) editions of the *EEPP* where Walras introduces a demand for a cash-balance (*encaisse desirée*) equation. As Rebeyrol (1999) notes it this stage of Walras's monetary theory clearly shows that he considers that money may be mainly thought as a medium of exchange and not as a medium to value transfers within two periods. However, this second stage did not explicitly deals with the actual exchange process.

From Lesson 29 of the fourth and fifth editions of the *EEPP* onwards, Walras's analysis is transformed in order to explicitly consider the exchange process. Up to this point, the theoretical *tâtonnement* process that leads to the formation of equilibrium prices of consumption goods, of capital and of land, takes place without any actual exchange.

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[9] For detailed discussion on the evolution of Walras's different versions of his monetary theory since the *EEPP* first edition till its final form presented in the fourth edition of this work see Marget (1931), Bridel (1997) and Rebeyrol (1999).

Tâtonnement is made on *bons* expressing the desired actions of agents. Through the first 28 lessons the main results take the form of existence theorems without any description of the passage from initial to final allocations. However, contrary to the tradition founded on the Arrow-Debreu model, Walras does not replace the exchange process by any centralised device as a clearing house or an active auctioneer delivering final allocations.

Walras's analysis of "circulation" takes us directly from the abstract world of pure theory to a highly practical reality. In fact, as has been underlined above, Walras considered money as a pure practical object not necessarily having its place within his pure economics. However, the evolution of his monetary theory shows how money becomes an integral part of his *EEPP* having its place as the closing device of his whole model. The introduction of a decentralised exchange process implies breaking with the centralised form of the (i.e. well organised) *tâtonnement sur bons*. Beyond this methodological change, let us explore the way Walras adds some novelties to the hypothesis of perfect competition framework in order to integrate his monetary theory.

3.2   The Technology of Exchanges and the Service of Availability of Money

The introduction of money leads to a new round of *tâtonnement* that allows adjusting the price of money as an availability service. According to Walras this adjustment implies a *tâtonnement* process that "should not seriously affect" the equilibrium on other markets[10]. Be this as it may, it is important to note that Walras concludes that the equilibrium of the monetary system exists. Money being a particular form of circulating capital it is enough to accept the coherence of the capitalisation model (section V of the *EEPP*, fifth edition) to ensure that the monetary model has a solution in which the demand for money is positive. In order to understand and comment this monetary theory it is necessary to look into its microeconomic grounds.

Walras introduces a concept taken from his theory of capital: the availability services (*services d'approvisionnement*). This concept is easily explained through the role of circulating capital in production. As producers cannot buy at each moment the raw materials they need to produce and as production takes time, a producer must try to buy a quantity of raw materials that exceed his immediate production needs. These stocks of raw materials

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[10] This loose remark leads Patinkin (1948) to accuse Walras to introduce the so-called 'invalid dichotomy'. However, as has been showed by Collard (1966) Walras's claim implies rather the exogeneity of the demand for money's service of availability. However, this also suggests that the Walrasian demand for money is founded on different grounds regarding the real part of the economy.

prove to be useful because they avoid the costs associated with buying a small quantity of raw materials at each moment of time. Hence, these stocks offer a necessary service to producers.

The circulation process (i.e. exchange) takes place during a unique logical period. Whose length is not fixed once and for all (all exchanges had to be completed on the market before another round of prices formation takes place).This implies that at the end of the *tâtonnement* agents have a period of time during which prices do not change and all exchanges respect the engagements made on the basis of the individual decisions (i.e. offer and demand) associated with those prices. At this point, Walras introduces the idea that entrepreneurs borrow money from consumers (workers, landowners and capitalists) (Walras 1988: 447). Money has no direct utility and is necessary for the realisation of exchanges due to a fundamental Walras's characterisation of the exchange process: the productive services workers, capitalists and landowners provide are supplied before entrepreneurs pay for them (Walras 1988: 441-2). Due to the asynchronies between deliveries and payments agents must keep a stock of money they will spend during the exchange period to buy goods, services and capitals. However, if entrepreneurs do not pay immediately for the services, why would they demand money? Walras's answer is: entrepreneurs need some money to buy raw materials (which are final products and not productive services). With their income, product of their sales, entrepreneurs pay at the end of the exchange period the value of the services of workers, capitalists and renters. According to Walras this ensures that at the end of the period agents will have the same sums of money they begun with.

3.3 The Difficulties of the Direct Exchange and the Perfect Competition Framework

Without discussing the well-founded of the technological problems associated with delivery and payment Walras introduces, we can underline three implicit points made by the description of the exchange process. The first one is that Walras describes a stationary situation where preferences and production technologies are unchanged and the amount of fixed capital is given. This is clearly stated in the paragraph 274 of the fourth edition of the *EEPP*. Thus exchange is not a time consuming process or, in other words, there is no positive rate of time discount within the actual period of exchanges. Walras is dealing with a logical and not a historical period of exchange. This will preclude all problems associated with modern 'search frictions' The Walrasian model is stated within a discrete time framework and the exchange process takes place within two periods. According to this, within the phase of exchange agents are indifferent to exchange at different "sub-periods". Contrary to what

Rebeyrol (1999: chapter 6) seems to assert, the time discount rate is positive only if one compares two periods, as it is the case in the Walras's capital theory.

Second, the exchange process Walras describes implicitly seems to take place through bilateral exchanges. This can be seen by observing Walras's reference to a double-coincidence problem in the traditional sense of term. It is, as a consequence of the division of labour, the specialisation in consumption and production. In the second edition of the *EEPP* a problem of "immediate" lack of double coincidence of wants is suggested:

> En effet, nous vendons nos services à des entrepreneurs qui ne fabriquent pas les produits dont nous avons besoin, et nous achetons des produits à des entrepreneurs qui n'emploient pas nos services. D'où la nécessité d'un intermédiaire d'échange…

(Walras 1988: 442)

If exchanges take place at equilibrium prices this implies agents may always fulfil their overall budget constraint at the end of the exchange process. However, if there is a double coincidence problem and exchanges take place sequentially in pairs through direct barter, as suggested by the works of Ostroy and Starr (Ostroy 1973 and Ostroy and Starr 1974), agents can not fulfil a condition of *quid pro quo* in every exchange. Only an indirect exchange can allow agents not having a double coincidence to exchange. This means that it is impossible to balance the equilibrium value of purchase and sale at every trading opportunity. As Rebeyrol correctly affirms (1999: 117-121 and 204-210) the presence of a numeraire does not guarantee the allocation of equilibrium quantities through a direct or indirect barter exchange process. We shall come back to this point further on. However, the *quid pro quo* constraint is not imposed on all the possible forms of exchange.

In fact, the third hypothesis Walras introduces is an asymmetric treatment of the different types of exchanges. Entrepreneurs can buy the productive services from owners-consumers without an immediate payment (Walras 1988: 443). These exchanges do not suppose the verification of an immediate *quid pro quo* constraint, even if agents may respect their overall budget constraint at the end of the exchange period (entrepreneurs must pay the productive services or the interests on the capital goods). Conversely, in order to buy finals goods, every agent must assure an immediate transfer of money (or goods) to the seller (Ibid: 441). This asymmetric treatment of exchanges implies that Walras introduces different forms of constraint according to different transactions. This raises a second form of the double coincidence problem which is now a temporal one: the desynchronise character of payments and deliveries.

From this, we can formulate a provisional conclusion: the main problem of direct barter Walras introduces in his monetary theory concerns the absence of double coincidence. And the desynchronised character of deliveries and payments is a particular source of the general problem, as suggested by the last quotation above. The actual realisation of exchanges trough a generalised direct barter exchange technology is precluded by the well-known problems associated to a specialised society. Moreover, Walras imposes a solution to the double coincidence problem through a particular form of cash-in-advance constraints that are active for some exchanges and do not bind for others.

Contrary to what is traditionally assumed within the neo-walrasian (à la Arrow-Debreu) framework, Walras does not assumed the centralised process of exchange though he assumes a centralised price formation process. This allows Walras to tackle with monetary questions from the medium of exchange point of view. We may assert that the 'store-of-value' approach to the integration of money and value theory, opened by Patinkin is not founded on Walras's theory of money. But it seems, as for the neo walrasian tradition, that the separation of the price-formation and the exchange process hinders Walras's theory of money. In fact, exchange takes place through a decentralised process after the well-organised *tâtonnement* has assured that the exchange rates respect the commutative justice condition.

4   An Appraisal of Walras's Attempt to Introduce Money within the Perfect Competition Framework

As Rebeyrol clearly states, Walras introduce the availability service of money in order to explain why agents are willing to exchange their goods for a good that has not direct utility for them :

> Walras took the idea that money is the instrument of exchange to its extreme limit. The problem of money has nothing to do with the intertemporal allocation of resources; it is linked entirely to the difficulties of carrying out transactions. It is for this reason, and not for its store of value function, that the cash balance is desired. (Rebeyrol 1998, 354)

On this, Walras shares the main question of Menger's monetary theory. Accepting this approach Walras has to explicitly detail the exchange process in order to justify that agents accept money because it allows realising the equilibrium final allocations. Otherwise, this final allocation is impossible to be reached. It is precisely in those terms that contemporary monetary theorists assert (Wallace 2002):

The notion of essentiality of money refers to the fact that "some good allocations are implementable using monetary trade that would otherwise not be implementable" (2).

However the essential character of money in Walras's theory is not completely proven. The problem here is to judge whether Walras was successful in integrating the use of money as a medium of exchange in his model of perfect competition, i.e. in his theory of value.

This question becomes even harder because the treatment of the "essentiality" of money takes different meanings depending on whether we follow Walras or Menger approach. In Menger's theory, as we show further on, the essentiality question implies two linked problems: first, the explicit comparison between different technologies of exchange and second, the choice of a particular object as the common medium of exchange. Whereas in Walras the main problem is to show how the environment of exchange explains why agents have interest in using a medium of exchange (i.e. justified the *encaisse désirée*). In any case, for both authors, the problem is not to deal with money as a store of value but as a medium of exchange.

4.1 The Choice of a Medium of Exchange

It is hard to find in Walras's work something indicating that he was interested in formulating a proposition on the process by which agents choose the particular commodity used as money. Hence, and as far as he did not change his mind, the following quote entails that he thought that this notion of integration falls into applied theory matters, as this quotation from the *EEPP* 2nd and 3rd editions shows:

> (…) Si des lois et décrets intervenaient pour donner cours légal et cours forcé à des cailloux ou à d'autres objets analogues, il est clair qu'au premier moment de trouble, l'autorité devenant impuissante, les détenteurs de cette monnaie seraient ruinés. Or, c'est précisément dans de telles circonstances que la monnaie doit avoir le plus de valeur (…) Mais c'est là une considération de théorie appliquée que nous pouvons écarter momentanément.
>
> (Walras 1988: 450-452)

Moreover, if he wanted to say something about this choice, he would have dealt with the following question: Why aren't the other circulating capitals universally accepted as means of exchange? Capitals also offer an availability service so there are capitals, such as low risk bonds, that may serve as means of exchange. One then can understand why Rebeyrol (1999) needs to suppose, contrary to Walras, that goods are not storable within the period of

exchange[11]. Otherwise, agents could buy all the needed quantities of goods for the period and the part of the service of availability of money that replaces the service of availability of other goods is futile. Following our interpretation, the positive demand for the particular monetary object (a fiat or metallic money) is *in fine* guaranteed by the hypothesis that it is the unique generally accepted medium of exchange.

Assuming money demand increases with the difficulties to exchange, we could say that Walras successfully incorporated money in his model as a result of a choice made by agents only if he can prove that 1/ agents have an interest in using money rather than any alternative transaction technology such as credit or barter, and 2/ that they have the ability to co-ordinate themselves on the use of this transaction technology. On this last point, it seems hard to imagine that Walras thought of some process of convergence of a population on money as this would be incoherent with the fact that he formulated his model in a static framework. We have then to conclude that the analysis of the monetisation of the economy was not in his goal. We can suppose that he thought it was an external/out-of-the-model decision. In other words, it seems difficult to assert that Walras succeeded a "real" integration of money into his value theory, as Bauvert (2002) already pointed out

We are left with the notion of integration of money theory into his value theory explained in terms of the essentiality of money. The solution to this problem can be dealt with using the results of one part of modern monetary theory. From the seventies on, some research in monetary economics concentrates on solving a pure theoretical problem consisting in understanding the conditions under which money is the solution to some of the problems agents face when exchanging. Gimenez (2002) and Bauvert (2002) have already remarked the analogy between a part of the modern theory of money (namely the search theoretic approach to money with fixed prices) and Walras's justification of the use of money. Here we shall go further into this analogy by using the principles stated by these recent advances in order to check whether the necessary assumptions made on the monetary part of the model are coherent with those made when dealing with price formation on the market[12]. Before this, we have to be perfectly clear about the way we use it.

Two different conceptions of money integration can be isolated. In the first conception, that we call "strategic stability", all agents have interest in using money once it had been established. This implies that the use of money has to be a dominant strategy

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[11] Rebeyrol (1999: 226-227) clearly shows the problem of the coexistence of money and other forms of service of availability in what he calls de D2.

[12] Note that this constitutes a variation of Wallace's (2001) argument whose aim was to check whether modern monetary theory can be used by macro models introducing money in a perfect equilibrium model.

compared to all alternative strategies of exchange (e.g. money has to dominate credit). If this is not the case, one can use a *strategic compatibility*[13] argument to ask whether the use of money could be permanent in that model. Following this notion of integration one has to check whether the assumptions made for money to dominate alternative ways of exchange are compatible with those required to have the perfect competition result. We do not want to discuss Walras using Menger's tools. We do not want to address critics to Walras based on another framework he did not want to deal with. We just want to emphasise that, for Walras to be successful in the integration of the use of money into his competition framework; we have to know something about the alternative exchange technology.

In the second notion of integration, that we shall call the "social habit", there is no concern with the fact that alternative ways to exchange are available or not and so with the fact that one can dominate the other. Hence, agents used money in any circumstance because the modeller has assumed so or because agents have some social habits that forgave them to question the return of this exchange technology. To put it another way, and in a more provocative fashion, agents do not apply economic theory to the choice of the exchange technology they used. In that interpretation, we don't have to require anything on the monetary part of Walras' model, the use of money is part of the fundamentals and the results of the competitive part has not to come into questions.

The latter interpretation refers to a weaker notion of integration of monetary theory into value theory than the former. In the next paragraph, we shall examine whether we can find room for the first notion of integration into Walras' model.

4.2   Exchange Technologies and the Essentiality of Money

As has been already showed, Walras supposes that money takes its value from the difficulties of exchanges and not from the direct utility it can provide. To rationalise his concept of availability service he postulates an environment in which agents face a problem of double coincidence of wants due to a lack of synchronisation between the moments in which agents are paid and those in which they want to spend. Money is then the instrument that allows agents to separate these two moments.

What are then the conditions under which there is a demand for money, once one particular object has been chosen to have this role? But notice that, following Marschak

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[13] The exchange strategies of an individual agent must be compatible with others' strategies of exchange in order to achieve the equilibrium final allocation: money is demanded because agents anticipate that other agents will implement a monetary strategy of exchange. This network effect of exchanges technologies is important in Walras's theory since he tries to explain the circulation of a useless object as the general medium of exchange.

(1949) and Hirshleifer (1972), in a situation where market-clearing prices have been established and where there is no uncertainty about the future possibilities of transaction all commodities are equally and perfectly "liquid". As is well known, the first assumption is fulfilled in Walras's theory where he explicitly assumes that agents know at the end of the *tâtonnement* the dates and prices of deliveries:

> La livraison des produits commencera de même immédiatement et continuera *d'une façon déterminée* pendant la même période. Le paiement de ces produits, évalués en numéraire, se fera aussi en monnaie à des *termes déterminés*.
>
> <div align="right">(Walras 1988: 441-443)</div>

This implies that there is no problem of solvability risk. We could however suppose that an idiosyncratic uncertainty concerning the exchange possibilities exists due to the lack of a synchronisation hypothesis. We shall distinguish between two situations: one in which every exchange must fulfil the quid pro quo condition (i.e. the value of the goods sold in one exchange has to be equal to the value of those bought in the same transaction) while the other will not.

Let us suppose a situation in which, as in Ostroy and Starr (1974), there are logistical problems associated with the exchange of goods. This is an economy in which prices are fixed on Sunday while exchanges are done throughout the rest of the week in decentralised manner. If a *quid pro quo* constraint is imposed for each bilateral exchange, there is no reason to suppose that in a decentralised process of exchange agents would follow the adequate pattern. Then, we understand that the existence of a useless general medium of exchange can overcome this problem[14]. And, as shown by Ostroy and Starr, the use of money yield a better result than indirect barter as barter can block the achievement of the process of goods' allocation.

However, it is not necessary for exchanges to be *quid pro quo* as it is the case when an entrepreneur buys a productive service which he pays latter. Then, without adding some form of credit, we can say nothing about the way exchanges will be conduct. Furthermore, if prices are equilibrium prices (and if agents know they are) and if agents respect their budget constraints within the period under consideration why don't they use a generalised credit system? Remember that if equilibrium prices result in a feasible allocation and the delivery and payments moments are determined, agents experiment no solvability risk by adopting an alternative exchange technology.

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[14] Rebeyrol (1999, chapter 6) advanced a similar argument in order to explain the necessity of a non-commodity medium of exchange in Walras's theory of exchange.

Walras also consider this case in which money is not the only conceivable technology of exchange that solves this kind of frictions. In fact, after having presented his monetary theory, Walras deals with two possible substitutes for money. In lesson 33 ("De la monnaie fiduciaire et des paiements par compensation", Walras 1988: 517-526) Walras clearly states that clearing houses, private credit, negotiable private bills, etc. actually substitute monetary exchange:

> Il y a, en effet, un certain nombre de moyens, dont l'importance se développe de jour en jour, de faire des échanges sans intervention de monnaie métallique. Ce sont les suivants : *crédit au livre … effet de commerce … billets de banque … chèques ...*

<div align="right">(Walras 1988 : 517)</div>

Introducing credit can deeply affect the terms of our previous answers. Consider the case in which there is no requirement in terms of *quid pro quo*. This will imply that on Monday, an agent will issue IOU, respecting the fixed equilibrium relative prices, in order to realise all exchanges. These IOU will be redeemable later, at the *determined* moment of delivery of agents' production. Given the supposed general equilibrium situation at the end of the week, we shall observe that agents have obtained their final allocations and all IOU have been redeemed. In that situation, each trade is not balanced at a particular point in time although there is an overall balancing of the exchange process. In that case, this decentralised (and temporal) credit system mimics the function performed by money. This corresponds to the argument use by Brunner and Meltzer (1971):

> If there is no costs of acquiring information, differences in the timing of receipts and payments are adjusted by issuing verbal promises in exchanges for goods and, later, delivering goods ... It is easy to see why 'lack of synchronisation' does not imply that money is used and held (785).

If a strict *quid pro quo* is not imposed, we can conceive that agents have the choice between being paid with good, money or through some credit arrangement. Taking into account the difficulties associated with barter, one can easily conclude there exists situations in which agents are indifferent between credit and money although they prefer both to barter. Hence, to prove that the use of money is in the interest of agents, one has to show that agents do not prefer to arrange their transactions through personal booking of individual debts rather than through money or barter.

Some recent works of the search theoretic approach to money can illuminate this point (see Wallace 2000 for a survey). All these works point out the necessity, for agents to prefer using money rather than IOU, that there have to be some imperfect knowledge of individual histories as this will allow agents to cheat their promises to repay and then give a role to a

medium of exchanges. That's only by lowering the return on the use of credit (i.e. the probability to be repaid), that this payment system credit lost its characteristic of perfect substitute to money.

Following the "strategic stability" approach, after having recognised the actual possibility of substitution between exchange technologies, we need to justify the superiority of monetary exchange. Walras increasingly searches for practical arguments in order to justify that *metallic money* is the most liquid of all mediums of payment. Forgetting the theoretical model supposes that exchanges are taking place at equilibrium prices and that there is not uncertainty[15], Walras states that private means of payment are necessary less liquid than money. But in order to state this "fact" (*constat*), Walras has to made use of traditional metallist argument extending his theory of fiat-money to a theory of metal-money (Lesson 32) and asserts that:

> Nous ne jugeons pas le fait ; nous *le constatons* et le précisons. En raison de leurs qualités exceptionnelles, l'or et l'argent sont de la *richesse liquide*. On peut les enfouir ou les déposer en lieu sûr avec la certitude qu'ils auront toujours leur valeur, et cela d'autant plus que les *circonstances seront plus critiques*. Des capitaux fixes ou circulants ne valent qu'en raison de la valeur de leur service ou de leur usage qui peut être nulle ou le devenir dans bien des cas. Donc les échanges réglés en titres ne sont pas liquidés.
>
> (Walras 1988: 521)

The fact that Walras considered the case in which credit and money can coexist seems to indicate that his model can include such a situation of imperfect monitoring. However, we have to check whether this assumption will not contradict those necessary for obtaining the competitive result in the price part of the model. But Walras is too vague for us to check this consistency as his use of the auctioneer prevents him to discuss deeply of the way one can get the "Walrasian outcome" on prices in a decentralised way. From the point of view of the "strategic stability" approach, Walras' theory of money seems, at least, incomplete.

5 Conclusion

The assumptions of perfect competition and the non strategic character of his model of price formation bind Walras. The institutional assumptions introduced in the theory of money have no grounds in what nowadays would be considered as the fundamentals of a walrassian

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[15] « Une légère incertitude à cet égard ne peut provenir que de la difficulté de prévoir les changements possibles dans les donnés du problème. En supposant ces données invariables pendant une certaine période de temps, et en supposant les prix des produits et des services, et leurs dates d'achat et de vente, connus pour toute cette période, nous ne laissons place à aucune incertitude. » (Walras 1988 : 443-445)

economy (i.e. preferences, technology and initial endowments). Walras is then forced to introduce the organisation of exchanges as part of the fundamentals. Given his demand for coherence and generality, this justification comes somewhat as a surprise. However, this leads to ask whether he simply supposes that agents use money because of "social habits" or an institutional constraint.

Nonetheless, two roads can now be set in motion. First, it could be that Walras's modification of his initial framework reveals that the naive character of agents in exchange and centralisation in the formation of prices must be abandoned if the use of money has to be explained. In that case, one has to go further to determine simultaneously the monetary pattern of exchanges and prices if we want to account for exchanges in a decentralised economy where strategic interactions among agents cannot be completely ignored. This leads us far from Walras's centralised theory of prices.

Second, it could be that Walras's monetary part is consistent with his price theory part. But this still has to be proved. To do so, one could use part of the recent theory of bargaining that tried, since the eighties, to micro-founded strategically the Walrasian outcome on prices. Note also, that challenging this view seems a little bit hard as he will faced the following difficulty, common in monetary theory, that we reproduce using Clower (1977) work:

> …essential sequence model [i.e. models in which money is essential] may turn out to be logically equivalent to 'non essential' sequence model of Arrow Debreu in which trading contract are concluded at just one instant in calendar time. The source of this equivalence is significant: it lies in the twin assumptions that traders are inhumanly prescient and that trading contracts and arrangements for future delivery of commodities can be negociated at zero cost… (208)

The solution Clower proposes to avoid this equivalence is contradictory with Walras's assumption on the deterministic delivery dates and payments:

> To drop the first assumptions, one has simply to suppose that individuals view future endowments as probable rather than certain while the second requirement can be dropped by assuming that individuals can negotiate trades only by engaging in extensive search and bargaining activities. (Ibid.)

This does not imply that Walras was wrong as it could be possible to avoid this equivalence in many ways. But this second route clearly seems to be a little bit sloping.

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